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PRINCETON UNIVERSITY

DEPARTMENT OF AEROSPACE AND MECHANICAL SCIENCES

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AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH

Status Report for the Period

1 January through 30 June 1968

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I. INTRODUCTION

During the past six months technical work of the ASMAR Program has continued as planned. Due to the fact that we were unable to obtain supplemental funds, the Program has been supported at the same level by University funds since mid-April. It will not be possible to continue this support during the summer months, however, and work during this normally productive period will be considerably reduced.

Dr. M. Handelsman will join the group on a permanent basis as a senior research scientist and lecturer, effective 1 July. It is planned that he present the Space Flight course next academic year while Professor L. Crocco is on leave.

II. SPACEFLIGHT TRAJECTORY ANALYSIS RESEARCH

A. Program Development

Documentation of the TOPCAT program has been completed and published as AMS Report No. 717s (see Reference 1). Several requests for this program have been received including McDonnell-Douglas Aircraft, Hughes Aircraft, and the University of Illinois.

The PRIMER program developed by Mr. Michael Minkoff has also been documented and will be issued shortly as Reference 2.

Testing of various modes of the TOMCAT (multi-body) program has been conducted. Two modes (N-body impulsive and patched conic impulsive) are working well. There are still problems with the low thrust multi-body mode. The program has been expanded to include three different types of Jupiter swingbys: solar probe (minimum perihelion), out-of-the-ecliptic (maximum inclination) and "galactic" probe (minimum time to 10 AU). Further development of the TOMCAT program will be incorporated within the more general framework of the trajectory compiler.

The basic modules for the trajectory compiler have been written in machine language for the IBM 360 computer. Compilation of the following trajectory segments has been demonstrated:

(1) LAUNCH: simulates ascent to parking orbit

(2) STATE: defines initial state vector

(3) IMPULSE: simulates impulsive burn

(4) INTEGRATE: integrates trajectory during thrust and coast period with multi-body perturbations

(5) TWO BODY: coast motion in inverse square field

(6) DROP MASS: calculates mass dropped after thrusting period

(7) REFERENCE CHANGE:

The compiler has been linked with the TOMCAT N-body and patched conic routines and the General Purpose Iterator for execution of the above trajectory segments.

Input and output variables are defined for each module. For example, for the launch module, required input variables include latitude and longitude of the launch site, time of launch, launch azimuth and altitude of parking orbit. Optional variables are, for example, time interval and arc length of the ascent arc. (For the present, it is intended that the ascent arc be simulated by Δt and $\Delta \emptyset$). Output variables would include state vector and time at the end of ascent. It will only be necessary for the analyst to select which of the input variables are constants and which are independent variables to be adjusted (automatically, by the General Purpose Iterator) to meet the constraints of the mission. The output variables from the LAUNCH module then become input to the next module selected by the programmer, together with any others as required. Standard values will be internal to the machine for those input variables which do not come from a previous module. It will be possible to override these by inputting any desired values.

Thus the various modules form a chain of computation with the output of one becoming the input of the next. The analyst selects only those links which he desired out of the variety of possible sequences. The General Purpose Iterator closes the computational loop around this chain, varying the selected independent variables so as to meet mission constraints (dependent variables). Output variables of any module in the sequence can be chosen as dependent (constrained) variables. If the analyst desires to constrain or input variables which are not included in the original formulation, this may be done by adding a small number of FORTRAN cards between the appropriate modules.

The trajectory compiler as outlined above is presently working in a preliminary state; that is, the program will work i input is correct. However, incorrect input gives unpredictable results which is a very severe problem in a program of this sort.

Test cases which have been checked so far using the trajectory compiler include:

1. Patch-conic interplanetary
   a. Jupiter flyby starting from earth launch
   b. Jupiter swingby starting from earth launch to get
      (1) Maximum out of the ecliptic
      (2) Minimum perihelion
      (3) Minimum time to 10 AU
2. Patch-conic lunar missions
   a. Lunar free return from launch pad
   b. Translunar trajectory from launch into polar lunar orbit
   c. Optimum three impulse lunar return from polar lunar orbit
3. Integrated multi-body mission
   a. Optimal low thrust transfers between coplanar circular orbits with
      (1) Minimum time
      (2) Minimum fuel

Work which remains to be done includes

1. Expansion of error recognition and checking.
2. Addition of first guess procedures and print options.
3. Documentation.

In addition, the basic trajectory modules will be expanded to include an orbital plane coordinate transformation and re-entry simulation.

B. Analytical Work

Mr. Michael Minkoff is pursuing the applications of multi-impulse trajectories using the PRIMER program which is now complete. He presented a seminar to the Flight Sciences Group of the Department of Aerospace and Mechanical Sciences last March. These results will be published as part of Mr. Minkoff's MSE thesis (Reference 2). In addition, a paper has been submitted to the XIX International Astronautical Congress which meets in New York this September.

Mr. Alain L. Kornhauser is continuing the investigation of the two variable expansion technique with a nonlinear time scale. The goal here is approximate analytical solutions for low thrust trajectories. This idea has been pursued with success in the case of linear time varying differential equations by Dr. R. Ramnath, a Princeton University graduate student (Reference 3). Mr. Kornhauser is applying this technique to nonlinear systems - in particular, the equations of motion in a gravitational field including thrust. An analysis of the existing literature applying linear time scales to this problem has been completed.

Another approach to the problem of approximate analytic solutions for finite thrust trajectories is being pursued jointly by Professor P. M. Lion and Mr. George A. Hazelrigg. This work began as an effort to find an analytic approximation for the (unknown) adjoint variables on the finite thrust trajectory in terms of the (known) adjoint variables on the corresponding impulsive trajectory; i.e., an extension to the "impulsive iterative" method (Reference 4).

In particular, an explicit relation has been derived in the form of a series in $1/a$. where a is the thrust acceleration. The zeroth order solution is the impulsive adjoint variables. Corrections are then calculated which improve the convergence properties of the impulsive iterative algorithm. In principle, this can be done to any order desired; in practice we have been restricted to first or second order in $1/a$ by algebraic intractability.

It is planned that the solution to this problem proceed in the following more or less logical manner:

(1) Constant thrust acceleration, linear equations of motion

(2) Constant thrust acceleration, nonlinear equations of motion

(3) Constant thrust, linear equation of motion

(4) Constant thrust, nonlinear equation of motion

The first two items above have already been developed and implemented on the computer (they will form a portion of the Ph.D. thesis of George A. Hazelrigg, Jr.). Thus feasibility of the technique is established. A sample of the results is shown in Figure 1. The case chosen was a two-burn transfer between circular orbits from $r = 1$ AU to $r = 1.5$ AU with a central travel angle of $3/4\pi$ and transit time of 3.3028 TAUS (200 days). Figure 1 shows the actual values of λ_1 and λ_2 versus a. The impulsive approximation (zeroth order), first order correction, and second order correction are each displayed separately.

Although this work directed toward solving the two-point boundary value problem is important in its own right, it now appears that the by-products will be of even greater significance. The series approximation just described makes possible an analytic description of the connection between finite thrust trajectories and the corresponding impulsive trajectories. In addition to the initial adjoint variables, the series approximation provides us with analytical estimates of "gravity losses" (i.e., propulsion penalty paid for


ACTUAL
SECOND ORDER
FIRST ORDER
λ_2
1.6
1.4
1.2
1.0
.8
.6
0
200
400
600 (s^2/m)
1
2
3
(n.u.)
$1/a$
CASE I, λ_2 SECOND ORDER
ACTUAL
SECOND ORDER
FIRST ORDER
λ_1
1.6
1.4
1.2
1.0
.8
.6
0
200
400
600 (s^2/m)
1
2
3
(n.u.)
$1/a$
TEST CASE I, λ_1 SECOND ORDER

FIGURE 1

finite thrusting time) and sensitivity coefficients (the change in trajectory characteristics for a small change in input parameters). For instance, to analyze a Mars mission one need only compute the optimum impulsive trajectory and then solve two systems of linear algebraic equations for the first and second order terms. The gravity losses can then be expressed in closed form for a wide range of propulsion parameters (a and VJ) i.e., for that range over which the series provide sufficient accuracy for mission analysis. From initial results it appears that this range definitely includes nuclear rocket propulsion systems and can be expected to include nuclear-electric systems with the inclusion of nonlinear terms which have so far not been included. It is not necessary to calculate any low thrust trajectories.

Dr. M. Handelsman, visiting senior research scientist, continued research on optimal, fixed-thrust elliptic-spiral trajectories in planetocentric fields. In addition, approximate solutions of elliptic spirals have been applied to the problem of optimal combinations of high and low-thrust propulsion for missions from earth to planetary orbit. There are no analytic solutions for optimal fixed-thrust spiral trajectories, excepting several special cases which consist of small changes in semi-major axis a and ellipticity e, or large changes only in a and orbital plane inclination i, for circular spirals only. For the broader problem of elliptic spirals, available optimal solutions assume propulsion with completely variable thrust magnitude. Variable-thrust solutions afford insight into the problem, and establish an upper bound to the payload performance of fixed-thrust engines, but can be unrealistic, and should be replaced by fixed-thrust solutions when possible.

The work to date concerns optimal spirals for large changes in a and e. The initial work has been the application of variable-thrust solutions to solar-electric missions for Mars orbiters, to establish an upper bound on mass performance for comparison with all-chemical-propulsion planetary maneuvers, discussed in the paragraph below. The remainder of the work concerns analytic solutions of the fixed-thrust case, and some trial numerical solutions.

For the Mars orbiter, the spacecraft arrives at Mars with mass M_{inc} and velocity V_{∞} . Electric power P and exhaust velocity V_{je} are (for the present) determined by already computed fuel-optimal fixed-time transfers from earth orbit, using TITAN launchers, solar-electric heliocentric propulsion, and a chemical retro into a specified circurlar orbit at Mars. In the new work, the spacecraft is first put into an elliptic orbit, with periapse radius r_p and ellipticity e , by a chemical retro-impulse with given I_{sp} at periapse, and then elliptically spiralled into the specified terminal circular orbit, using the solar-electric engine. The initial chemical retro ΔV , the retro propellant, and the spacecraft mass into initial elliptic orbit are then calculable. The initial electric fixed-thrust acceleration A_o is

$$A_o = F/M_o = 2\eta P/V_{je} \tag{1}$$

where F = thrust, and engine efficiency η depends upon V_{je} . For variable thrust, the characteristic velocity of the optimal ellipse-to-circle spiral is

$$\overline{A}t = f(V_c, r_p, e) \tag{2}$$

where

$\overline{A}$ = thrust acceleration <u>averaged</u> over a spiral turn, and is a constant from the variational theory

t = transfer time

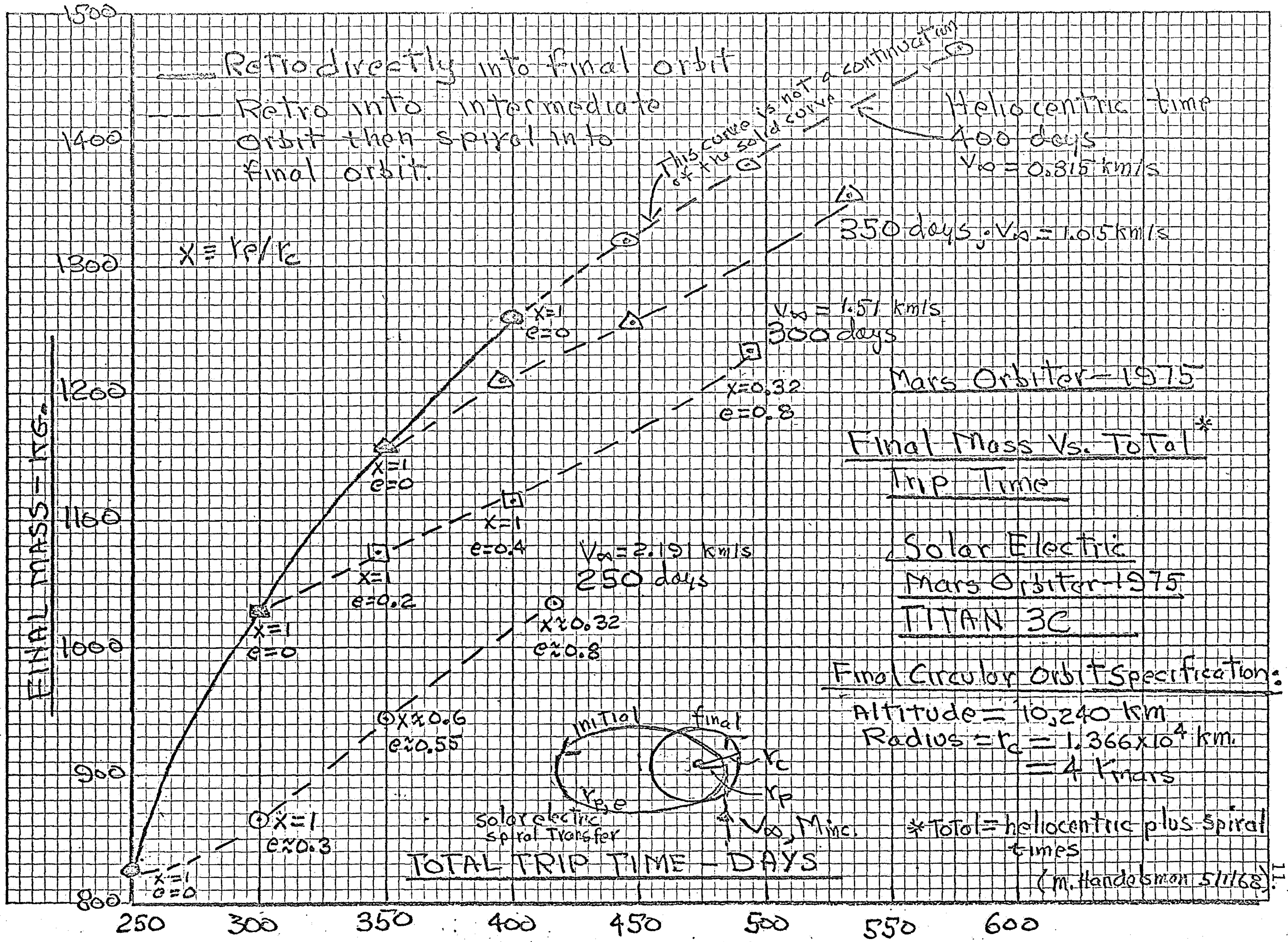
V_c = final orbit circular velocity

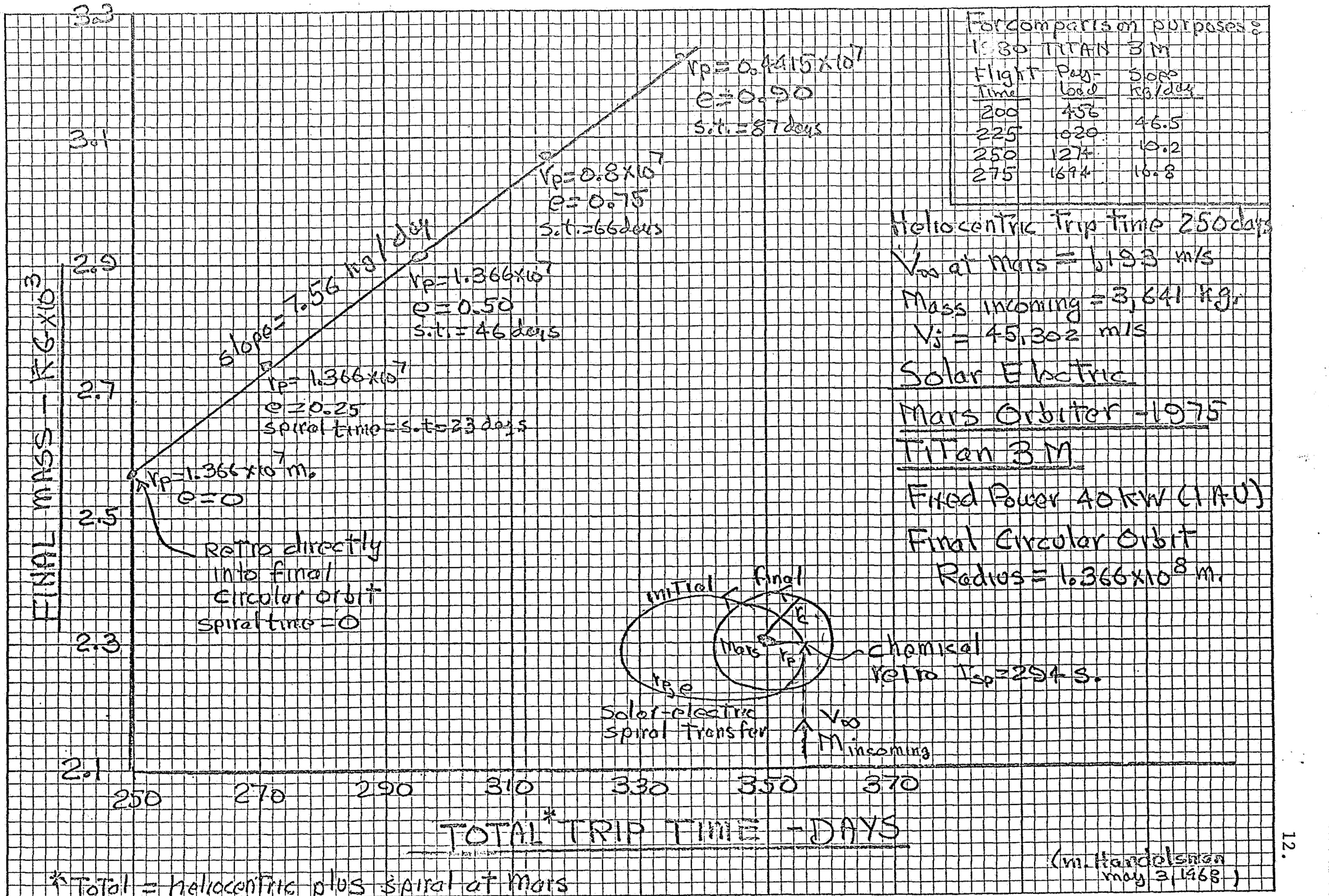
r_p, e = periapse radius and ellipticity of initial orbit

An upper bound for the fixed-thrust final mass is available by assuming that A_o , Eq. (1) and $\bar{A}$, Eq. (2), are the same. While $\bar{A}$ is a constant, the optimal instantaneous variable thrust magnitude changes with orbital position and orbital elements. The decrease in performance involved by replacing the variable thrust with fixed-thrust is unknown, and is the problem under attack. Variable thrust affords an additional degree of freedom over fixed thrust, and therefore furnishes an upper performance bound. A second-order improvement in accuracy is possible by using a mean fixed-thrust acceleration averaged over the maneuver, calculable by a simple iterative formula, instead of the initial acceleration A_o . However, for the maneuvers considered herein, the mass loss is small, and the fixed-thrust acceleration changes relatively little from its initial value. Assuming that $\bar{A} = A_o$, the fixed-thrust spiral time is

$$t = \frac{1}{A_o} f(V_c, r_p, e) \tag{3}$$

This time t is a minimum value; the actual fixed-thrust time will exceed this value. The spiral propellant mass and final mass into terminal circular orbit are then calculable. These equations have been programmed for a final circular orbit of 4 Mars radii. Typical results are shown in the two attached graphs, titled TITAN 3C and TITAN 3M, Mars Orbiter, 1975, Solar Electric. It is seen that there is a very considerable fuel savings over the chemical retro directly into final circular orbit. If the planetocentric spiral time is exchanged for heliocentric transit time, it turns out that the mass savings


Retro directly into final orbit
Retro into intermediate orbit then spiral into final orbit.
X = rp/rc
This curve is not a continuation of the solid curve
Heliocentric time 400 days
V∞ = 0.815 km/s
350 days, V∞ = 1.05 km/s
V∞ = 1.51 km/s
300 days
V∞ = 2.19 km/s
250 days
X=1 e=0
X=1 e=0.4
X=1 e=0.2
X=0.32 e=0.8
X≈0.32 e≈0.8
X≈0.6 e≈0.55
X=1 e≈0.3
Initial
final
rc
rp
rp,e
solar electric spiral Transfer
V∞, Minc.
Mars Orbiter – 1975
Final Mass Vs. Total* Trip Time
Solar Electric
Mars Orbiter-1975
TITAN 3C
Final Circular Orbit Specification:
Altitude = 10,240 km
Radius = rc = 1.366x10^4 km.
= 4 rmars
*Total = heliocentric plus spiral times
(M. Handelsman 5/1/68)
FINAL MASS - KG.
TOTAL TRIP TIME - DAYS
800
900
1000
1100
1200
1300
1400
1500
250
300
350
400
450
500
550
600


For comparison purposes:
1980 TITAN 3M
Flight Time | Pay-load | Slope kg/day
200 | 456 |
225 | 620 | 46.5
250 | 1274 | 10.2
275 | 1694 | 16.8
Heliocentric Trip time 250 days
V∞ at Mars = 1193 m/s
Mass incoming = 3,641 kg.
Vj = 45,302 m/s
Solar Electric
Mars Orbiter - 1975
Titan 3M
Fixed Power 40 KW (1 AU)
Final Circular Orbit
Radius = 1.366x10^8 m.
rp = 0.4415x10^7
e = 0.90
s.t. = 87 days
rp = 0.8x10^7
e = 0.75
s.t. = 66 days
rp = 1.366x10^7
e = 0.50
s.t. = 46 days
slope = 7.56 kg/day
rp = 1.366x10^7
e = 0.25
spiral time = s.t = 23 days
rp = 1.366x10^7 m.
e = 0
Retro directly into final circular orbit spiral time = 0
initial
final
rc
Mars
rp
rp, e
chemical retro Isp = 294 s.
Solar-electric spiral transfer
V∞
M incoming
FINAL MASS - KG x10^3
3.3
3.1
2.9
2.7
2.5
2.3
2.1
250
270
290
310
330
350
370
TOTAL* TRIP TIME - DAYS
*Total = heliocentric plus spiral at Mars
(M. Handelsman May 3, 1968)

due to increased heliocentric time is larger than the savings afforded by the spiral maneuver. However, this is not a fair general conclusion, because these are particular results, based upon available trajectories optimized for a single retro transfer directly into a circular orbit. A fair comparison requires trajectories optimized for the complete transfer, including an intermediate planetary spiral. In addition, for planetary scientific observations and experiments, a given planetocentric maneuver time interval may be more "valuable" than an equal heliocentric time interval, and should be so considered in defining a desirable optimum mission pay-off.

The work on optimal solutions to the fixed-thrust elliptic-spiral case is divided into analytic and numeric approaches. The analytic work further divides into two approaches. The first is a variational calculus approach using orbital elements as state variables. Here progress has been made in obtaining some closed-form analytic results, and this is now being intensively pursued and extended. The other analytic approach, based upon Hamilton-Jacobi perturbation theory, is now being studied. Either of these two approaches may be used to obtain approximate analytic solutions which are of considerable value for first-order mission planning analysis, and for furnishing good initial Lagrangian multiplier values for rapid iterative computer solutions.

A limited numerical investigation of optimal fixed-thrust spirals has been started. These are for transfers between circular orbits in earth's field. The complete optimal variational equations in a central body field are used. The initial Lagrangian multipliers were estimated from a quasi-optimal circular spiral solution. The available TOMCAT program was used, with simplifications appropriate to this problem to reduce running time.

Both minimum time (continuous propulsion) and minimum propellant (coast periods) cases have been run, with results given in the Table below. These are preliminary results to test the application of an available program to this problem, and to form an estimate of required running times, accuracy, etc. The improvement in circularity of final orbit with iteration is the main feature of interest (i.e., compare cases 2 to 3 and 5 to 6).

Table of Low-Thrust Spirals

	Trajectory Type	Final Radius	Approx. No. of Turns	Iteration Used or Not	Ellipticity of Final Orbit
1.	Min. Fuel	4 R_{EARTH}	120	No	0.02
2.	Min. Fuel	5 R_E	8	No	0.64
3.	Min. Fuel	5 R_E	8	Yes	0.07
4.	Min. Time	1.6 R_E	25	No	0.02
5.	Min. Time	5 R_E	4	No	0.42
6.	Min. Time	5 R_E	4	Yes	0.01

III. AEROSPACE SYSTEMS ANALYSIS RESEARCH

The staff engaged in the ASAR program in the first two quarters of 1968 were Dr. R. Vichnevetsky (leading the program), Mr. M. D. Mintz (consultant), Messrs. C. F. Kalmbach and R. J. Chin (undergraduate students), and Mrs. A. B. Shulzycki (programmer).

The work of the ASAR Group in this period has resulted in the completion of a mathematical model for nuclear rocket engines. This model has been implemented on the 7094 digital computer as a computer code (NUROCSAC). This code is organized in such a manner that it can be used for the representation of the hot bleed as well as the topping cycle. Engine system studies have been performed by the use of this computer code by analyzing the influence of design parameters on engine mass, on engine specific performance as well as on mission related criteria.

For this purpose, a general theory of mission related sensitivity functions has been developed, which permits the optimization of engine parameters with respect to mission related objectives (such as payload or initial mass in earth orbit). In essence, mission related engine parameters sensitivity and optimization are achieved by independently running computer programs for the determination of mission related sensitivity functions, and then using these as inputs to engine systems analysis programs for the calculation of mission related engine performance. This method is one of staged analysis and optimization.

Results obtained to date in the ASAR program in the first six months of 1968 by use of this method have indicated a considerable potential in savings of computer utilization time over the computer programs achieving the same results by a simultaneous computation of engine and trajectory equations.

One of the detailed analyses in the program was devoted to a study of the effect of nuclear rocket chamber pressure upon engine mass as well as payload for a one-way Mars mission.

A course on Advanced Methods of Systems Analysis, which has been developed over the past three years as an adjunct to the ASAR program, has been presented this spring as a graduate course in the Department by Dr. R. Vichnevetsky and has drawn attendance from most departments within the Engineering School.

Results obtained by the ASAR program in the first six months of 1968 have been reported in the following memoranda and reports.

ASAR Memo No. 10, Pressure Drop Calculation for Nuclear Rocket System Engineering, M. D. Mintz, 15 December 1967.

ASAR Memo No. 11, External Sensitivity Analysis of Nuclear Rocket Engineering Model, M. D. Mintz, 15 December 1967.

ASAR Memo No. 12, Turbopump Bleed Fractions and Thrust Calculations for Nuclear Rocket Engines, R. Vichnevetsky, 24 January 1968.

ASAR Memo No. 13, Sizing the Nuclear Rocket Core, R. Vichnevetsky, 21 February 1968.

ASAR Memo No. 14, Systems Analysis of Nuclear Rocket Engines, R. Vichnevetsky, M. D. Mintz and C. F. Kalmbach, 6 April 1968.

Vichnevetsky, R., Mintz, M. D. and Kalmbach, C. F., Systems Analysis of Nuclear Rocket Engines, Princeton University AMS Report No. 717y, 6 April 1968.

References

1. Lion, P. M., Campbell, J. H. and Shulzycki, A. B., Trajectory Optimization Program for Comparing Advanced Technologies, TOPCAT I, Princeton University, Department of Aerospace and Mechanical Sciences, ASMAR Program, Aerospace Engineering Report No. 717s, March 1, 1968.

2. Minkoff, M., Optimal Multi-Impulse Space Trajectories, MSE Thesis, Princeton University, June 1968.

3. Ramnath, R., A Multiple Time Scales Approach to the Analysis of Linear Systems, Ph.D. Thesis, Princeton University, 1967.

4. Handelsman, M., Optimal Free-Space Fixed-Thrust Trajectories Using Impulsive Trajectories as Starting Iteratives, AIAA J., 4, No. 6, June 1966, 1077-1082.